

Package ‘MLMES’

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Type Package

Title Model-Based Effect Sizes for Multilevel Models

Version 0.1.2

Description Computes model-based effect sizes for fixed-effect coefficients in multilevel (hierarchical) models. The coefficient effect sizes are standardized mean differences from zero (d) and unique variance-explained measures (squared semi-partial correlations, sr2). The package also reports variance components and level-specific and total R-squared values. It supports 2-level and 3-level linear and binary logistic models fitted with 'lme4' (Bates et al., 2015) <doi:10.18637/jss.v067.i01>, and 2-level Gaussian and Bernoulli models fitted with 'brms' (Bürkner, 2017) <doi:10.18637/jss.v080.i01>. Sanders, Konold, and Cheng (in press), ``Model-based effect sizes for multilevel linear regression coefficients," Methodology: European Journal of Research Methods for the Behavioral and Social Sciences, describe the 2-level linear-model methods.

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Contents

achieve_3L_data	2
glmerES_3L	3
glmerRsq_3L	5
glmerVarComp_3L	6
hsb_data	8
lmerES	10
lmerES_3L	11
lmerRsq	13
lmerRsq_3L	15
lmerVarComp	16
lmerVarComp_3L	17
MLMES_brms	19
MLMES_brms_ci	21
MLMES_glmer	23
readgrowth_data	25
Index	26

achieve_3L_data	<i>Three-Level Student Achievement Dataset</i>
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Description

A processed subset of the Achieve data used in Chapter 4, "Level-3 and Higher Models," of Finch, Bolin, and Kelley (2024). The data contain 10,320 students (L1) nested within 568 classrooms (L2), which are nested within 160 schools (L3).

Usage

```
data("achieve_3L_data")
```

Format

- A data frame with 10,320 observations on 9 variables:
- gread** General reading achievement score (outcome)
 - z_gevocab_CMC** General vocabulary score, classroom-mean-centered and standardized (L1)
 - gender_eff** Effect-coded gender: original code 1 = -1 and original code 2 = 1 (L1)
 - z_gemath_agg_CMC** Classroom mean mathematics achievement, school-mean-centered and standardized (L2)
 - z_ptratio_CMC** Pupil-teacher ratio, school-mean-centered and standardized (L2)
 - z_ses_GMC** School socioeconomic status, grand-mean-centered and standardized (L3)
 - school** School identifier (L3 unit)
 - class** Classroom code within school
 - school_class** Unique classroom identifier (L2 unit)

Details

Continuous predictors were centered according to their model level and then standardized. The student vocabulary score was centered within classrooms; classroom mean mathematics achievement and pupil-teacher ratio were centered within schools; and school socioeconomic status was centered at the grand mean. Only the variables used in the package examples are retained.

Source

<https://www.mlminr.com/data-sets>

References

Finch, W. H., Bolin, J. E., & Kelley, K. (2024). *Multilevel modeling using R* (3rd ed.). Chapman & Hall/CRC. doi:10.1201/b23166

glmerES_3L	<i>Coefficient Effect Sizes for 3-Level Binary Logistic Multilevel Models</i>
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Description

Computes the standardized mean difference from zero (d) and unique variance-explained (sr^2) for each fixed-effect coefficient in a 3-level binary logistic model fitted with lme4::glmer. Effect sizes are based on latent variance.

Usage

```
glmerES_3L(model, verbose = TRUE)
```

Arguments

- | | |
|---------|--|
| model | A 3-level model fitted with lme4::glmer, using the binomial family and logit link. |
| verbose | Logical; if TRUE (default), prints a note explaining column abbreviations. |

Details

The standardized mean difference and squared semi-partial correlation follow the definitions used for linear models, but their denominators are based on latent variance. L1 predictors should be centered within L2 units, and L2 predictors should be centered within L3 units. Continuous predictors should be z-scored after centering, and categorical predictors should be effect-coded.

Value

A data frame with coefficient estimates and model-based effect sizes:

Variable Predictor name.

Level Predictor level: 1, 2, or 3.

RS_L2 Random-slope status at L2: 1 = yes, 0 = no.

RS_L3 Random-slope status at L3: 1 = yes, 0 = no.

Coeff Unstandardized logit coefficient.

SE Standard error.

z z statistic.

p p value.

d_lev Standardized mean difference using level-specific variance.

d_L1_L2 Standardized mean difference using L1 and L2 variance; reported for L1 predictors.

d_L2_L3 Standardized mean difference using L2 and L3 variance; reported for L2 predictors.

d_tot Standardized mean difference using total variance.

sr2_lev Unique proportion of level-specific variance explained.

sr2_L1_L2 Unique proportion of L1 and L2 variance explained; reported for L1 predictors.

sr2_L2_L3 Unique proportion of L2 and L3 variance explained; reported for L2 predictors.

sr2_tot Unique proportion of total variance explained.

References

Sanders, E. A., Konold, T. R., & Cheng, Y. (in press). Model-based effect sizes for multilevel linear regression coefficients. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences*.

McCulloch, C. E., Searle, S. R., & Neuhaus, J. M. (2008). *Generalized, linear, and mixed models* (2nd ed.). Wiley.

Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software*, 67(1), 1–48. doi:10.18637/jss.v067.i01

See Also

[glmerES](#) for 2-level models, [glmerVarComp_3L](#), [glmerRsqr_3L](#)

Examples

```
data("achieve_3L_data")

cutvalue_read <- stats::quantile(achieve_3L_data$gread, probs = 0.40)
achieve_binary <- transform(
  achieve_3L_data,
  gread_pass = as.integer(gread >= cutvalue_read)
)
```

```

three_level_logit_fit <- lme4::glmer(
  geread_pass ~ z_gevocab_CMC + gender_eff +
    z_gemath_agg_CMC + z_ptratio_CMC + z_ses_GMC +
    (1 | school_class) + (1 | school),
  data = achieve_binary,
  family = binomial,
  control = lme4::glmerControl(
    optimizer = "bobyqa",
    optCtrl = list(maxfun = 2e5)
  )
)

glmerES_3L(three_level_logit_fit)

```

glmerRsq_3L

*R-Squared Measures for 3-Level Binary Logistic Multilevel Models***Description**

Returns latent-scale R-squared values for the variance sources in a 3-level binary logistic model fitted with `lme4::glmer`.

Usage

```
glmerRsq_3L(model, verbose = TRUE)
```

Arguments

<code>model</code>	A 3-level model fitted with <code>lme4::glmer</code> , using the binomial family and logit link.
<code>verbose</code>	Logical; if TRUE (default), prints a note explaining column abbreviations.

Details

Rows beginning with FE, L1_RS, L2_RS, or RI give R-squared values for the indicated variance source. Rows containing total combine fixed- and random-effects variance sources across levels.

Value

A data frame with R-squared values for each variance source or combination of sources:

Effect Variance source or combination of sources.

TotalRsq Proportion of total latent variance explained.

L1Rsq Proportion of L1 latent variance explained.

L2Rsq Proportion of L2 latent variance explained.

L3Rsq Proportion of L3 latent variance explained.

References

- Sanders, E. A., Konold, T. R., & Cheng, Y. (in press). Model-based effect sizes for multilevel linear regression coefficients. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences*.
- McCulloch, C. E., Searle, S. R., & Neuhaus, J. M. (2008). *Generalized, linear, and mixed models* (2nd ed.). Wiley.
- Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software*, 67(1), 1–48. doi:10.18637/jss.v067.i01

See Also

[glmerRsq](#) for 2-level models, [glmerVarComp_3L](#), [glmerES_3L](#)

Examples

```
data("achieve_3L_data")

cutvalue_read <- stats::quantile(achieve_3L_data$gread, probs = 0.40)
achieve_binary <- transform(
  achieve_3L_data,
  gread_pass = as.integer(gread >= cutvalue_read)
)

three_level_logit_fit <- lme4::glmer(
  gread_pass ~ z_gevocab_CMC + gender_eff +
    z_gemath_agg_CMC + z_ptratio_CMC + z_ses_GMC +
    (1 | school_class) + (1 | school),
  data = achieve_binary,
  family = binomial,
  control = lme4::glmerControl(
    optimizer = "bobyqa",
    optCtrl = list(maxfun = 2e5)
  )
)

glmerRsq_3L(three_level_logit_fit)
```

glmerVarComp_3L

Variance Components for 3-Level Binary Logistic Multilevel Models

Description

Returns the latent-scale variance components used to calculate model-based effect sizes for a 3-level binary logistic model fitted with `lme4::glmer`.

Usage

```
glmerVarComp_3L(model, verbose = TRUE)
```

Arguments

<code>model</code>	A 3-level model fitted with <code>lme4::glmer</code> , using the binomial family and logit link.
<code>verbose</code>	Logical; if TRUE (default), prints a note explaining column abbreviations.

Details

The mean response probability is approximated from the model intercept and random-effects variance following McCulloch, Searle, and Neuhaus (2008). Observation-level variance is then calculated as $1/\{\bar{p}(1 - \bar{p})\}$.

L1 predictors should be centered within L2 units, and L2 predictors should be centered within L3 units. Continuous predictors should be z-scored after centering, and categorical predictors should be effect-coded.

Value

A data frame with rows for L1, L2, L3, and all variance sources:

levels Variance level: L1, L2, L3, or All.

FE_Var Fixed-effects variance.

L1_Resid_Var Observation-level variance.

L1_RS_L2_Var Variance from L1 slopes varying across L2 units.

L1_RS_L3_Var Variance from L1 slopes varying across L3 units. When an L1 and an L2 predictor both have random slopes at L3, their slope contributions covary, and the full L3 slope variance exceeds the sum of the two separate variances by twice that covariance. This component includes one copy of the covariance and L2_RS_Var includes the other, so the two together account for the full L3 slope variance. Under the cluster-mean centering the framework assumes, the covariance is exactly zero and this term is the plain variance.

L2_RI_Var Conditional L2 random-intercept variance.

L2_RS_Var Variance from L2 slopes varying across L3 units. Includes the second copy of the covariance described under L1_RS_L3_Var.

L3_RI_Var Conditional L3 random-intercept variance.

TotVar Variance for the specified level.

SD Square root of TotVar.

References

- Sanders, E. A., Konold, T. R., & Cheng, Y. (in press). Model-based effect sizes for multilevel linear regression coefficients. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences*.
- McCulloch, C. E., Searle, S. R., & Neuhaus, J. M. (2008). *Generalized, linear, and mixed models* (2nd ed.). Wiley.
- Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software*, 67(1), 1–48. doi:10.18637/jss.v067.i01

See Also

[glmerVarComp](#) for 2-level models, [glmerRsq_3L](#), [glmerES_3L](#)

Examples

```
data("achieve_3L_data")

cutvalue_read <- stats::quantile(achieve_3L_data$geread, probs = 0.40)
achieve_binary <- transform(
  achieve_3L_data,
  geread_pass = as.integer(geread >= cutvalue_read)
)

three_level_logit_fit <- lme4::glmer(
  geread_pass ~ z_gevocab_CMC + gender_eff +
    z_gemath_agg_CMC + z_ptratio_CMC + z_ses_GMC +
    (1 | school_class) + (1 | school),
  data = achieve_binary,
  family = binomial,
  control = lme4::glmerControl(
    optimizer = "bobyqa",
    optCtrl = list(maxfun = 2e5)
  )
)

glmerVarComp_3L(three_level_logit_fit)
```

hsb_data

High School and Beyond (HSB) Dataset

Description

Cross-sectional data on math achievement and sociodemographic characteristics for 7,185 students from 160 U.S. high schools. Students (L1) are nested within schools (L2). The data include the original variables and the centered, effect-coded, and z-scored predictors used in the package examples.

Usage

```
data("hsb_data")
```

Format

A data frame with 7,185 observations on 27 variables:

school School identifier (level-2 unit)

size School size

sector School sector (1 = Catholic, 0 = public)

disclim Disciplinary climate

himinty Indicator for more than 40 percent minority enrollment (1 = yes, 0 = no)

student Student identifier

minority Minority status (1 = yes, 0 = no)

female Female indicator (1 = yes, 0 = no)

ses Socioeconomic status

mathach Mathematics achievement score (outcome)

minority_eff Minority status, effect-coded

female_eff Female indicator, effect-coded

minority_eff_CMC Minority status, effect-coded and cluster-mean-centered (L1)

female_eff_CMC Female indicator, effect-coded and cluster-mean-centered (L1)

ses_CMC SES, cluster-mean-centered (L1)

z_ses_CMC SES, cluster-mean-centered and z-scored (L1)

mathach_agg School-aggregated math achievement

ses_agg School-aggregated SES

sqrt_size Square root of school size

sector_eff School sector, effect-coded (L2)

himinty_eff High minority enrollment, effect-coded (L2)

ses_agg_GMC Aggregated SES, grand-mean-centered (L2)

sqrt_size_GMC Square root of school size, grand-mean-centered (L2)

disclim_GMC Disciplinary climate, grand-mean-centered (L2)

z_ses_agg_GMC Aggregated SES, grand-mean-centered and z-scored (L2)

z_sqrt_size_GMC Square root of school size, grand-mean-centered and z-scored (L2)

z_disclim_GMC Disciplinary climate, grand-mean-centered and z-scored (L2)

References

Raudenbush, S. W., & Bryk, A. S. (2002). *Hierarchical linear models: Applications and data analysis methods* (2nd ed.). Thousand Oaks, CA: Sage Publications.

lmerES

*Coefficient Effect Sizes for 2-Level Linear Multilevel Models***Description**

Computes two model-based effect sizes for each fixed-effect coefficient in a 2-level hierarchical linear model fitted with `lme4::lmer`: the standardized mean difference from zero (d) and unique variance-explained (sr^2). Each effect size is reported relative to level-specific variance and total variance.

Usage

```
lmerES(model, verbose = TRUE)
```

Arguments

<code>model</code>	A 2-level linear model fitted with <code>lme4::lmer</code> .
<code>verbose</code>	Logical; if TRUE (default), prints a note explaining column abbreviations.

Details

The standardized mean difference expresses a coefficient relative to the model-implied standard deviation of the outcome. The squared semi-partial correlation gives the proportion of outcome variance uniquely explained by a predictor after accounting for its shared variance with the other predictors. Level-specific effect sizes use variance sources associated with the predictor level; total effect sizes use all L1 and L2 variance sources.

Degrees of freedom and p values are taken from `summary(model)`. Fit the model with `lmerTest::lmer`, or convert an existing `lme4::lmer` model with `lmerTest::as_lmerModLmerTest`, to report these values. For an `lme4::lmer` model, the `df` and `p` columns contain NA.

L1 predictors should be cluster-mean-centered. Continuous L1 and L2 predictors should be z-scored after centering, and categorical predictors should be effect-coded. When the model is fitted by maximum likelihood (REML = FALSE), finite-sample bias corrections are applied before the effect sizes are calculated.

Value

A data frame with coefficient estimates and model-based effect sizes:

Variable Predictor name.

Level Predictor level: 1 or 2.

RS Random-slope status: 1 = yes, 0 = no.

Coeff Unstandardized coefficient estimate.

SE Standard error.

t t statistic.

df Degrees of freedom, when available.

p p value, when available.
d_lev Standardized mean difference using level-specific variance.
d_tot Standardized mean difference using total variance.
sr2_lev Unique proportion of level-specific variance explained.
sr2_tot Unique proportion of total variance explained.

References

Sanders, E. A., Konold, T. R., & Cheng, Y. (in press). Model-based effect sizes for multilevel linear regression coefficients. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences*.

Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software*, 67(1), 1–48. doi:10.18637/jss.v067.i01

Kuznetsova, A., Brockhoff, P. B., & Christensen, R. H. B. (2017). lmerTest package: Tests in linear mixed effects models. *Journal of Statistical Software*, 82(13), 1–26. doi:10.18637/jss.v082.i13

See Also

[lmerES_3L](#) for 3-level models, [lmerVarComp](#), [lmerRsq](#)

Examples

```
data("hsb_data")

if (requireNamespace("lmerTest", quietly = TRUE)) {
  hsb_m1c <- lmerTest::lmer(mathach ~
    minority_eff_CMC + female_eff_CMC + z_ses_CMC +
    z_ses_agg_GMC +
    z_sqrt_size_GMC + sector_eff + z_disclim_GMC + himinty_eff +
    (1 + minority_eff_CMC + female_eff_CMC + z_ses_CMC | school),
    data = hsb_data, REML = FALSE,
    control = lme4::lmerControl(optimizer = "bobyqa",
                                optCtrl = list(maxfun = 2e5)))

  lmerES(hsb_m1c)
}
```

Description

Computes two model-based effect sizes for each fixed-effect coefficient in a 3-level hierarchical linear model fitted with `lme4::lmer`: the standardized mean difference from zero (d) and unique variance-explained (sr^2). Effect sizes are reported relative to level-specific, adjacent-level, and total variance where applicable.

Usage

```
lmerES_3L(model, verbose = TRUE)
```

Arguments

<code>model</code>	A 3-level linear model fitted with <code>lme4::lmer</code> .
<code>verbose</code>	Logical; if TRUE (default), prints a note explaining column abbreviations.

Details

The standardized mean difference expresses a coefficient relative to the model-implied standard deviation of the outcome. The squared semi-partial correlation gives the proportion of outcome variance uniquely explained by a predictor after accounting for its shared variance with the other predictors.

Degrees of freedom and p values are taken from `summary(model)`. Fit the model with `lmerTest::lmer`, or convert an existing `lme4::lmer` model with `lmerTest::as_lmerModLmerTest`, to report these values. For an `lme4::lmer` model, the `df` and `p` columns contain NA.

L1 predictors should be centered within L2 units, and L2 predictors should be centered within L3 units. Continuous predictors should be z-scored after centering, and categorical predictors should be effect-coded. When the model is fitted by maximum likelihood (REML = FALSE), finite-sample bias corrections are applied before the effect sizes are calculated.

Value

A data frame with coefficient estimates and model-based effect sizes:

Variable Predictor name.

Level Predictor level: 1, 2, or 3.

RS_L2 Random-slope status at L2: 1 = yes, 0 = no.

RS_L3 Random-slope status at L3: 1 = yes, 0 = no.

Coeff Unstandardized coefficient estimate.

SE Standard error.

t t statistic.

df Degrees of freedom, when available.

p p value, when available.

d_lev Standardized mean difference using level-specific variance.

d_L1_L2 Standardized mean difference using L1 and L2 variance; reported for L1 predictors.

d_L2_L3 Standardized mean difference using L2 and L3 variance; reported for L2 predictors.

d_tot Standardized mean difference using total variance.

sr2_lev Unique proportion of level-specific variance explained.

sr2_L1_L2 Unique proportion of L1 and L2 variance explained; reported for L1 predictors.

sr2_L2_L3 Unique proportion of L2 and L3 variance explained; reported for L2 predictors.

sr2_tot Unique proportion of total variance explained.

References

- Sanders, E. A., Konold, T. R., & Cheng, Y. (in press). Model-based effect sizes for multilevel linear regression coefficients. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences*.
- Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software*, 67(1), 1–48. doi:10.18637/jss.v067.i01
- Kuznetsova, A., Brockhoff, P. B., & Christensen, R. H. B. (2017). lmerTest package: Tests in linear mixed effects models. *Journal of Statistical Software*, 82(13), 1–26. doi:10.18637/jss.v082.i13

See Also

[lmerES](#) for 2-level models, [lmerVarComp_3L](#), [lmerRsq_3L](#)

Examples

```
data("achieve_3L_data")

three_level_fit <- lmerTest::lmer(
  geread ~ z_gevocab_CMC + gender_eff +
    z_gemath_agg_CMC + z_ptratio_CMC + z_ses_GMC +
    (1 | school_class) + (1 | school),
  data = achieve_3L_data,
  REML = FALSE
)

lmerES_3L(three_level_fit)
```

lmerRsq

R-Squared Measures for 2-Level Linear Multilevel Models

Description

Returns R-squared values for the variance sources in a 2-level hierarchical linear model fitted with lme4::lmer. Level-specific values use the variance sources associated with L1 or L2; total values use all L1 and L2 variance sources.

Usage

```
lmerRsq(model, verbose = TRUE)
```

Arguments

model	A 2-level linear model fitted with lme4::lmer.
verbose	Logical; if TRUE (default), prints a note explaining column abbreviations.

Details

Rows beginning with FE_L1, FE_L2, RS_L1, or RI_L2 give R-squared values for a single variance source. Rows beginning with FE_total, FE_RS_total, or FE_RS_RI_total combine the indicated fixed- and random-effects variance sources.

Value

A data frame with R-squared values for each variance source or combination of sources:

Effect Variance source or combination of sources.

TotalRsq Proportion of total variance explained.

L1Rsq Proportion of L1 variance explained.

L2Rsq Proportion of L2 variance explained.

References

Sanders, E. A., Konold, T. R., & Cheng, Y. (in press). Model-based effect sizes for multilevel linear regression coefficients. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences*.

Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software*, 67(1), 1–48. doi:10.18637/jss.v067.i01

See Also

[lmerRsq_3L](#) for 3-level models, [lmerVarComp](#), [lmerES](#)

Examples

```
data("hsb_data")

hsb_m1c <- lme4::lmer(mathach ~
  minority_eff_CMC + female_eff_CMC + z_ses_CMC +
  z_ses_agg_GMC +
  z_sqrt_size_GMC + sector_eff + z_disclim_GMC + himinty_eff +
  (1 + minority_eff_CMC + female_eff_CMC + z_ses_CMC | school),
  data = hsb_data, REML = FALSE,
  control = lme4::lmerControl(optimizer = "bobyqa",
    optCtrl = list(maxfun = 2e5)))

lmerRsq(hsb_m1c)
```

lmerRsq_3L

*R-Squared Measures for 3-Level Linear Multilevel Models***Description**

Returns R-squared values for the variance sources in a 3-level hierarchical linear model fitted with lme4::lmer. Level-specific values use the variance sources associated with L1, L2, or L3; total values use all three levels.

Usage

```
lmerRsq_3L(model, verbose = TRUE)
```

Arguments

model A 3-level linear model fitted with lme4::lmer.
verbose Logical; if TRUE (default), prints a note explaining column abbreviations.

Details

Rows beginning with FE, L1_RS, L2_RS, or RI give R-squared values for the indicated variance source. Rows containing total combine fixed- and random-effects variance sources across levels.

Value

A data frame with R-squared values for each variance source or combination of sources:

Effect Variance source or combination of sources.

TotalRsq Proportion of total variance explained.

L1Rsq Proportion of L1 variance explained.

L2Rsq Proportion of L2 variance explained.

L3Rsq Proportion of L3 variance explained.

References

Sanders, E. A., Konold, T. R., & Cheng, Y. (in press). Model-based effect sizes for multilevel linear regression coefficients. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences*.

Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software*, 67(1), 1–48. doi:10.18637/jss.v067.i01

See Also

[lmerRsq](#) for 2-level models, [lmerVarComp_3L](#), [lmerES_3L](#)

Examples

```
data("achieve_3L_data")

three_level_fit <- lme4::lmer(
  geread ~ z_gevocab_CMC + gender_eff +
    z_gemath_agg_CMC + z_ptratio_CMC + z_ses_GMC +
    (1 | school_class) + (1 | school),
  data = achieve_3L_data,
  REML = FALSE
)

lmerRsqr_3L(three_level_fit)
```

lmerVarComp

*Variance Components for 2-Level Linear Multilevel Models***Description**

Returns the variance components used to calculate model-based effect sizes for a 2-level hierarchical linear model fitted with `lme4::lmer`. The components are L1 and L2 fixed-effects variance, L1 random-slopes variance, conditional L2 random-intercept variance, and residual L1 variance.

Usage

```
lmerVarComp(model, verbose = TRUE)
```

Arguments

model A 2-level linear model fitted with `lme4::lmer`.

verbose Logical; if TRUE (default), prints a note explaining column abbreviations.

Details

L1 predictors should be cluster-mean-centered. Continuous L1 and L2 predictors should be z-scored after centering, and categorical predictors should be effect-coded. When the model is fitted by maximum likelihood (`REML = FALSE`), finite-sample bias corrections are applied to the random-effects and residual variance estimates.

Value

A data frame with rows for L1, L2, and all variance sources:

level Variance level: L1, L2, or All.

FE_Var Fixed-effects variance.

L1_Resid_Var Residual L1 variance.

L1_RS_Var L1 random-slopes variance.

- L2_RI_Var** Conditional L2 random-intercept variance.
- TotVar** Variance for the specified level.
- SD** Square root of TotVar.

References

Sanders, E. A., Konold, T. R., & Cheng, Y. (in press). Model-based effect sizes for multilevel linear regression coefficients. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences*.

Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software*, 67(1), 1–48. doi:10.18637/jss.v067.i01

See Also

[lmerVarComp_3L](#) for 3-level models, [lmerRsq](#), [lmerES](#)

Examples

```
data("hsb_data")

hsb_m1c <- lme4::lmer(mathach ~
  minority_eff_CMC + female_eff_CMC + z_ses_CMC +
  z_ses_agg_GMC +
  z_sqrt_size_GMC + sector_eff + z_disclim_GMC + himinty_eff +
  (1 + minority_eff_CMC + female_eff_CMC + z_ses_CMC | school),
  data = hsb_data, REML = FALSE,
  control = lme4::lmerControl(optimizer = "bobyqa",
    optCtrl = list(maxfun = 2e5)))

lmerVarComp(hsb_m1c)
```

<code>lmerVarComp_3L</code>	<i>Variance Components for 3-Level Linear Multilevel Models</i>
-----------------------------	---

Description

Returns the variance components used to calculate model-based effect sizes for a 3-level hierarchical linear model fitted with `lme4::lmer`. Components are reported for L1, L2, L3, and all variance sources.

Usage

```
lmerVarComp_3L(model, verbose = TRUE)
```

Arguments

- `model` A 3-level linear model fitted with `lme4::lmer`.
- `verbose` Logical; if TRUE (default), prints a note explaining column abbreviations.

Details

This function extends the 2-level variance decomposition described by Sanders, Konold, and Cheng (in press) to three levels.

L1 predictors should be centered within L2 units, and L2 predictors should be centered within L3 units. Continuous predictors should be z-scored after centering, and categorical predictors should be effect-coded. When the model is fitted by maximum likelihood (REML = FALSE), finite-sample bias corrections are applied to the random-effects and residual variance estimates.

Value

A data frame with rows for L1, L2, L3, and all variance sources:

levels Variance level: L1, L2, L3, or All.

FE_Var Fixed-effects variance.

L1_Resid_Var Residual L1 variance.

L1_RS_L2_Var Variance from L1 slopes varying across L2 units.

L1_RS_L3_Var Variance from L1 slopes varying across L3 units. When an L1 and an L2 predictor both have random slopes at L3, their slope contributions covary, and the full L3 slope variance exceeds the sum of the two separate variances by twice that covariance. This component includes one copy of the covariance and L2_RS_Var includes the other, so the two together account for the full L3 slope variance. Under the cluster-mean centering the framework assumes, the covariance is exactly zero and this term is the plain variance.

L2_RI_Var Conditional L2 random-intercept variance.

L2_RS_Var Variance from L2 slopes varying across L3 units. Includes the second copy of the covariance described under L1_RS_L3_Var.

L3_RI_Var Conditional L3 random-intercept variance.

TotVar Variance for the specified level.

SD Square root of TotVar.

References

Sanders, E. A., Konold, T. R., & Cheng, Y. (in press). Model-based effect sizes for multilevel linear regression coefficients. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences*.

Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software*, 67(1), 1–48. doi:10.18637/jss.v067.i01

See Also

[lmerVarComp](#) for 2-level models, [lmerRsqr_3L](#), [lmerES_3L](#)

Examples

```
data("achieve_3L_data")

three_level_fit <- lme4::lmer(
  geread ~ z_gevocab_CMC + gender_eff +
    z_gemath_agg_CMC + z_ptratio_CMC + z_ses_GMC +
    (1 | school_class) + (1 | school),
  data = achieve_3L_data,
  REML = FALSE
)

lmerVarComp_3L(three_level_fit)
```

MLMES_brms

*Effect Sizes from 2-Level Bayesian Multilevel Models***Description**

Computes model-based coefficient effect sizes, variance components, and R-squared values from posterior summaries of a 2-level model fitted with `brms::brm`.

Usage

```
brms_lmerVarComp(model, robust = TRUE, probs = 0.95, verbose = TRUE)

brms_lmerRsq(model, robust = TRUE, probs = 0.95, verbose = TRUE)

brms_lmerES(model, robust = TRUE, method = "HDI", probs = 0.95, verbose = TRUE)
```

Arguments

<code>model</code>	A 2-level Gaussian or Bernoulli model fitted with <code>brms::brm</code> .
<code>robust</code>	Logical; if TRUE (default), posterior locations are summarized by the median and posterior scales by the median absolute deviation.
<code>probs</code>	Numeric scalar giving the posterior interval probability. Default is 0.95.
<code>verbose</code>	Logical; if TRUE (default), prints a note explaining column abbreviations.
<code>method</code>	Character; method for computing credible intervals, passed to <code>bayestestR::ci</code> . Used for the coefficient intervals returned by <code>brms_lmerES</code> . Default is "HDI".

Details

These functions apply the 2-level variance-decomposition calculations to posterior point summaries. Use `brms_lmerES_ci`, `brms_lmerVarComp_ci`, or `brms_lmerRsq_ci` for intervals calculated from posterior draws.

For Bernoulli models, the effect sizes and variance components are on the latent scale. Observation-level variance is based on the mean response probability approximation described by McCulloch, Searle, and Neuhaus (2008).

Predictors should be centered, z-scored, and effect-coded as described in [lmerES](#). All functions require **brms**; **brms_lmerES** also requires **bayestestR**.

Value

brms_lmerES returns coefficient posterior summaries, coefficient interval bounds, R-hat, and the columns `d_lev`, `d_tot`, `sr2_lev`, and `sr2_tot`. The effect sizes are calculated from posterior point summaries.

brms_lmerVarComp returns rows for L1, L2, and all variance sources, with columns `level`, `L1_FE_Var`, `L1_Resid_Var`, `L1_RS_Var`, `L2_RI_Var`, `TotVar`, and `SD`.

brms_lmerRsq returns `Effect`, `TotalRsq`, `L1Rsq`, and `L2Rsq`. R-squared values are proportions calculated from posterior point summaries.

References

Sanders, E. A., Konold, T. R., & Cheng, Y. (in press). Model-based effect sizes for multilevel linear regression coefficients. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences*.

Bürkner, P.-C. (2017). brms: An R package for Bayesian multilevel models using Stan. *Journal of Statistical Software*, 80(1), 1–28. doi:10.18637/jss.v080.i01

Makowski, D., Ben-Shachar, M. S., & Lüdtke, D. (2019). bayestestR: Describing effects and their uncertainty, existence and significance within the Bayesian framework. *Journal of Open Source Software*, 4(40), 1541. doi:10.21105/joss.01541

McCulloch, C. E., Searle, S. R., & Neuhaus, J. M. (2008). *Generalized, linear, and mixed models* (2nd ed.). Wiley.

See Also

[brms_lmerES_ci](#), [brms_lmerVarComp_ci](#), [brms_lmerRsq_ci](#) for versions with credible intervals; [lmerES](#) for frequentist 2-level models.

Examples

```
if (requireNamespace("brms", quietly = TRUE) &&
    requireNamespace("bayestestR", quietly = TRUE)) {
  data("hsb_data")

  hsb_brm_m1c <- brms::brm(mathach ~
    minority_eff_CMC + female_eff_CMC + z_ses_CMC +
    z_ses_agg_GMC +
    z_sqrt_size_GMC + sector_eff + z_disclim_GMC + himinty_eff +
    (1 + minority_eff_CMC + female_eff_CMC + z_ses_CMC | school),
    data = hsb_data, family = gaussian(),
    chains = 2, iter = 2000, warmup = 1000, cores = 2, seed = 212931)

  brms_lmerVarComp(hsb_brm_m1c)
```

```

    brms_lmerES(hsb_brm_m1c)
    brms_lmerRsq(hsb_brm_m1c)
  }

```

MLMES_brms_ci

Posterior Intervals for Effect Sizes from 2-Level Bayesian Models

Description

Computes posterior intervals for model-based coefficient effect sizes, variance components, and R-squared values from a 2-level model fitted with `brms::brm`.

Usage

```

brms_lmerVarComp_ci(
  model,
  robust = TRUE,
  method = "HDI",
  probs = 0.95,
  verbose = TRUE
)

brms_lmerRsq_ci(
  model,
  robust = TRUE,
  method = "HDI",
  probs = 0.95,
  verbose = TRUE
)

brms_lmerES_ci(
  model,
  robust = TRUE,
  method = "HDI",
  probs = 0.95,
  verbose = TRUE
)

```

Arguments

<code>model</code>	A 2-level Gaussian or Bernoulli model fitted with <code>brms::brm</code> .
<code>robust</code>	Logical; passed to the brms posterior summaries used during model setup. Default is TRUE.
<code>method</code>	Character; method for computing credible intervals, passed to <code>bayestestR::ci</code> . Default is "HDI".

probs	Numeric; probability for credible intervals. Default is 0.95.
verbose	Logical; if TRUE (default), prints a note explaining column abbreviations.

Details

Intervals are calculated from posterior draws of the corresponding quantities; the returned tables do not include point estimates. For Bernoulli models, the quantities are on the latent scale.

Predictors should be centered, z-scored, and effect-coded as described in [lmerES](#). These functions require **brms** and **bayestestR**.

Value

`brms_lmerES_ci` returns Variable, Level, and RS, followed by lower and upper posterior interval bounds for `d_lev`, `d_tot`, `sr2_lev`, and `sr2_tot`.

`brms_lmerVarComp_ci` returns rows for L1, L2, and all variance sources, with lower and upper posterior interval bounds for each variance component, TotVar, and SD.

`brms_lmerRsq_ci` returns Effect and lower and upper posterior interval bounds for TotalRsq, L1Rsq, and L2Rsq. R-squared values are proportions.

References

- Sanders, E. A., Konold, T. R., & Cheng, Y. (in press). Model-based effect sizes for multilevel linear regression coefficients. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences*.
- Bürkner, P.-C. (2017). brms: An R package for Bayesian multilevel models using Stan. *Journal of Statistical Software*, 80(1), 1–28. doi:10.18637/jss.v080.i01
- Makowski, D., Ben-Shachar, M. S., & Lüdtke, D. (2019). bayestestR: Describing effects and their uncertainty, existence and significance within the Bayesian framework. *Journal of Open Source Software*, 4(40), 1541. doi:10.21105/joss.01541
- McCulloch, C. E., Searle, S. R., & Neuhaus, J. M. (2008). *Generalized, linear, and mixed models* (2nd ed.). Wiley.

See Also

[brms_lmerES](#), [brms_lmerVarComp](#), [brms_lmerRsq](#) for versions without credible intervals.

Examples

```
if (requireNamespace("brms", quietly = TRUE) &&
    requireNamespace("bayestestR", quietly = TRUE)) {
  data("hsb_data")

  hsb_brm_m1c <- brms::brm(mathach ~
    minority_eff_CMC + female_eff_CMC + z_ses_CMC +
    z_ses_agg_GMC +
    z_sqrt_size_GMC + sector_eff + z_disclim_GMC + himinty_eff +
    (1 + minority_eff_CMC + female_eff_CMC + z_ses_CMC | school),
    data = hsb_data, family = gaussian(),
```

```

chains = 2, iter = 2000, warmup = 1000, cores = 2, seed = 212931)

brms_lmerVarComp_ci(hsb_brm_m1c)
brms_lmerES_ci(hsb_brm_m1c)
brms_lmerRsq_ci(hsb_brm_m1c)
}

```

MLMES_glmer

*Effect Sizes for 2-Level Binary Logistic Multilevel Models***Description**

Computes model-based coefficient effect sizes, variance components, and R-squared values for a 2-level binary logistic model fitted with `lme4::glmer`. The calculations use a latent-scale variance decomposition.

Usage

```

glmerES(model, verbose = TRUE)

glmerVarComp(model, verbose = TRUE)

glmerRsq(model, verbose = TRUE)

```

Arguments

<code>model</code>	A fitted model object from <code>lme4::glmer</code> with 2-level structure and binomial family with logit link.
<code>verbose</code>	Logical; if TRUE (default), prints a note explaining column abbreviations.

Details

The standardized mean difference and squared semi-partial correlation follow the definitions used for linear models, but their denominators are based on latent variance. The mean response probability is approximated from the model intercept and random-effects variance following McCulloch, Searle, and Neuhaus (2008). Observation-level variance is then calculated as $1/\{\bar{p}(1 - \bar{p})\}$.

L1 predictors should be cluster-mean-centered. Continuous L1 and L2 predictors should be z-scored after centering, and categorical predictors should be effect-coded.

Value

`glmerES` returns a data frame with the following columns:

Variable Predictor name.

Level Predictor level: 1 or 2.

RS Random-slope status: 1 = yes, 0 = no.

Coeff Unstandardized logit coefficient.

SE Standard error.

z z statistic.

p p value.

d_lev, d_tot Standardized mean differences using level-specific and total latent variance.

sr2_lev, sr2_tot Unique proportions of level-specific and total latent variance explained.

glmerVarComp returns rows for L1, L2, and all variance sources, with columns level, L1_FE_Var, L1_Resid_Var, L1_RS_Var, L2_RI_Var, TotVar, and SD.

glmerRsqr returns Effect, TotalRsqr, L1Rsqr, and L2Rsqr. R-squared values are proportions.

References

Sanders, E. A., Konold, T. R., & Cheng, Y. (in press). Model-based effect sizes for multilevel linear regression coefficients. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences*.

McCulloch, C. E., Searle, S. R., & Neuhaus, J. M. (2008). *Generalized, linear, and mixed models* (2nd ed.). Wiley.

Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software*, 67(1), 1–48. doi:10.18637/jss.v067.i01

See Also

[glmerES_3L](#), [glmerVarComp_3L](#), [glmerRsqr_3L](#) for 3-level models; [lmerES](#), [lmerVarComp](#), [lmerRsqr](#) for linear models.

Examples

```
data("hsb_data")

cutvalue_math <- quantile(hsb_data$mathach, probs = 0.40)
hsb_data$mathach_pass <- ifelse(hsb_data$mathach >= cutvalue_math, 1, 0)

hsb_glm_m1c <- lme4::glmer(mathach_pass ~
  minority_eff_CMC + female_eff_CMC + z_ses_CMC +
  z_ses_agg_GMC +
  z_sqrt_size_GMC + sector_eff + z_disclim_GMC + himinty_eff +
  (1 + minority_eff_CMC + female_eff_CMC + z_ses_CMC | school),
  data = hsb_data,
  family = binomial,
  control = lme4::glmerControl(optimizer = "bobyqa",
                                optCtrl = list(maxfun = 2e5)))

glmerES(hsb_glm_m1c)
glmerVarComp(hsb_glm_m1c)
glmerRsqr(hsb_glm_m1c)
```

readgrowth_data	<i>Early Reading Growth Dataset</i>
-----------------	-------------------------------------

Description

Longitudinal data from 79 children who received an individual, phonics-based reading intervention in first grade. Children were assessed at four waves over three years, beginning in the fall of first grade. Repeated observations (L1) are nested within children (L2), and the data are in long format.

Usage

```
data("readgrowth_data")
```

Format

A data frame with 316 observations on 18 variables:

schnum2 School identifier

subnum2 Child identifier (level-2 unit)

ppvty1_1 Receptive vocabulary, Peabody Picture Vocabulary Test standard score

rlny1_1 Rapid letter naming, Comprehensive Test of Phonological Processing raw score

yoppy1_1 Phonemic awareness, Yopp-Singer Test of Phoneme Segmentation raw score

ppvt_GMC Receptive vocabulary (PPVT), grand-mean-centered (L2)

rln_GMC Rapid letter naming, grand-mean-centered (L2)

yopp_GMC Phonemic awareness (Yopp-Singer), grand-mean-centered (L2)

z_ppvt_GMC Receptive vocabulary (PPVT), grand-mean-centered and z-scored (L2)

z_rln_GMC Rapid letter naming, grand-mean-centered and z-scored (L2)

z_yopp_GMC Phonemic awareness (Yopp-Singer), grand-mean-centered and z-scored (L2)

time Time point

WR Woodcock-Johnson Mastery Test Word Identification raw score (outcome)

Decode Decoding score

Spell Spelling score

TimeLin Linear time regressor

Rate1 Piecewise rate regressor for growth during first grade (L1)

Rate2 Piecewise rate regressor for growth during grades 2 and 3 (L1)

References

Vadasy, P. F., Sanders, E. A., & Abbott, R. D. (2008). Effects of supplemental early reading intervention at 2-year follow up: Reading skill growth patterns and predictors. *Scientific Studies of Reading*, 12(1), 51–89. doi:10.1080/10888430701746906

Index

* datasets

achieve_3L_data, [2](#)
hsb_data, [8](#)
readgrowth_data, [25](#)

achieve_3L_data, [2](#)

brms_lmerES, [22](#)
brms_lmerES (MLMES_brms), [19](#)
brms_lmerES_ci, [19](#), [20](#)
brms_lmerES_ci (MLMES_brms_ci), [21](#)
brms_lmerRsq, [22](#)
brms_lmerRsq (MLMES_brms), [19](#)
brms_lmerRsq_ci, [19](#), [20](#)
brms_lmerRsq_ci (MLMES_brms_ci), [21](#)
brms_lmerVarComp, [22](#)
brms_lmerVarComp (MLMES_brms), [19](#)
brms_lmerVarComp_ci, [19](#), [20](#)
brms_lmerVarComp_ci (MLMES_brms_ci), [21](#)

glmerES, [4](#)
glmerES (MLMES_glmer), [23](#)
glmerES_3L, [3](#), [6](#), [8](#), [24](#)
glmerRsq, [6](#)
glmerRsq (MLMES_glmer), [23](#)
glmerRsq_3L, [4](#), [5](#), [8](#), [24](#)
glmerVarComp, [8](#)
glmerVarComp (MLMES_glmer), [23](#)
glmerVarComp_3L, [4](#), [6](#), [6](#), [24](#)

hsb_data, [8](#)

lmerES, [10](#), [13](#), [14](#), [17](#), [20](#), [22](#), [24](#)
lmerES_3L, [11](#), [11](#), [15](#), [18](#)
lmerRsq, [11](#), [13](#), [15](#), [17](#), [24](#)
lmerRsq_3L, [13](#), [14](#), [15](#), [18](#)
lmerVarComp, [11](#), [14](#), [16](#), [18](#), [24](#)
lmerVarComp_3L, [13](#), [15](#), [17](#), [17](#)

MLMES_brms, [19](#)
MLMES_brms_ci, [21](#)

MLMES_glmer, [23](#)

readgrowth_data, [25](#)